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ON THE COMPUTATION OF THE POSITION IN ORBIT OF A PARTICLE IN THE TAIL OF A COMET.

By PROF. ORMOND STONE, University of Virginia.

We have the well known equations

$$f_{,}^2 = k^2 \left(\frac{2}{r_{,}} - \frac{1}{a} \right), \tag{1}$$

$$f_{,} \cos \beta_{,} = \frac{k}{\sqrt{p}} e \sin v_{,} \tag{2}$$

$$f_{,} \sin \beta_{,} = \frac{k\sqrt{p}}{r_{,}}; \tag{3}$$

where, neglecting the mass of the comet,

k is the constant of the solar system,

e, a, p, the eccentricity, semi major axis, and semi parameter of the orbit of the comet,

$r_{,}$, $v_{,}$, the radius vector and true anomaly for any given time $t_{,}$,

$f_{,}$, the corresponding velocity in orbit, and

$\beta_{,}$, the angle which the radius vector makes with the tangent.

(2) and (3) give $$\tan \beta_{,} = \frac{p}{r_{,}} \cdot \frac{1}{e \sin v_{,}} = \frac{1 + e \cos v_{,}}{e \sin v_{,}}. \tag{a}$$

For a free particle moving with the comet up to the time $t_{,}$, but after that moving in a hyperbolic orbit subject to a repulsive force k^2x acting in addition to gravity, but in opposition thereto, we have

$$f^2 = k^2 (1 - x) \left(\frac{2}{R} + \frac{1}{b} \right), \tag{4}$$

$$f \cos \beta = \frac{k\sqrt{(1 - x)}}{\sqrt{P}} E \sin V, \tag{5}$$

$$f \sin \beta = \frac{k\sqrt{(1 - x)}\sqrt{P}}{R}; \tag{6}$$

in which b is the hyperbolic semi-major axis.

(1) and (4) give $$\frac{2}{r_{,}} - \frac{1}{a} = (1 - x) \left(\frac{2}{r_{,}} + \frac{1}{b} \right),$$

since at the time $t_,$, $f = f_,$ and $R = r_,$;

$$\therefore\ b = \frac{1-x}{2x} \cdot \frac{r_,}{1 - r_, / 2ax}, \qquad (b)$$

$$= \frac{1-x}{2x} r_,, \text{ when the orbit of the comet is a parabola.}$$

Putting $E = \sec\psi$, whence $P = b\tan^2\psi$, (1) and (6) give

$$f_, \sin\beta_, = k\sqrt{\left(\frac{2}{r_,} - \frac{1}{a}\right)} \cdot \sin\beta_, = k\sqrt{(1-x)}\frac{\sqrt{b}}{r_,}\tan\psi;$$

whence, substituting the value of b given in (b) and solving, we obtain

$$\tan\psi = \frac{2\sqrt{x}}{1-x} \cdot \sqrt{\left(1 - \frac{r_,}{2a}\right)} \cdot \sqrt{\left(1 - \frac{r_,}{2ax}\right)} \cdot \sin\beta_,, \qquad (c)$$

$$= \frac{2\sqrt{x}}{1-x}\sin\beta_,, \text{ when the orbit of the comet is a parabola.}$$

The sign of E is assumed the same as that of $\tan\psi$, hence negative when $x > 1$.

(5) and (6) give
$$\tan\beta = \frac{P}{E} \cdot \frac{1}{R\sin V}, \qquad (7)$$

where, if F be the hyperbolic anomaly,

$$R\sin V = b\tan\psi\tan F.$$

Substituting in (7) the values of E, P, and $R\sin V$, and simplifying, we have

$$\tan F = \sin\psi\cot\beta, \qquad (d)$$

or, for the time $t_,$,
$$\tan F_, = \sin\psi\cot\beta_,. \qquad (d_,)$$

After computing the formulæ (a), (b), (c), and $(d_,)$, $r_,$ and $V_,$ are found from the well known equations

$$r_, \sin V_, = b\tan\psi\tan F_,,$$
$$r_, \cos V_, = b(E - \sec F_,).$$

The agreement of this value of $r_,$ with that derived directly from the orbit of the comet will form a convenient check.

The mean motion and time of hyperbolic perihelion passage t_0 is found from the equations

$$n = \frac{\lambda k\sqrt{(1-x)}}{b^{\frac{3}{2}}},$$

$$N_, = n(t_, - t_0) = \lambda E\tan F_, - \log\tan(45^\circ + \tfrac{1}{2}F_,),$$

where λ is the modulus of common logarithms.

The position in orbit for any given time t is found from the formulæ

$$\begin{aligned} \lambda E \tan F - \log \tan (45^\circ + \tfrac{1}{2}F) &= n(t - t_0) = N, \\ R \sin V &= b \tan \psi \tan F, \\ R \cos V &= b(E - \sec F). \end{aligned} \qquad (e)$$

F is found from the first of these equations by successive approximation. If $E \sec F$ be negative or be positive and large, $1/(E \sec F - 1)$ will be small and we may obtain $\tan F_1$ from the equation

$$\tan F_1 = \frac{N + \log \tan (45^\circ + \frac{1}{2}F_0)}{\lambda E},$$

where F_0 is an assumed value of F. Subtracting this from

$$\tan F = \frac{N + \log \tan (45^\circ + \frac{1}{2}F)}{\lambda E},$$

we have, including only terms of the first order,

$$F = F_1 + \frac{F_1 - F_0}{E \sec F - 1}.$$

If F be not too large we may express the first of equations (e) in the form

$$E \cdot \tfrac{1}{2}(e^x - e^{-x}) - x = n_0(t - t_0) = N_0,$$

where $$\lambda x = \log \tan (45^\circ + \tfrac{1}{2}F)$$

and $$n_0 = \frac{k\sqrt{(1 - x)}}{b^{\frac{3}{2}}}.$$

Since $$\frac{e^x - e^{-x}}{2} = \frac{x}{1} + \frac{x^3}{1 \cdot 2 \cdot 3} + \frac{x^5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} + \ldots,$$

if we put $$\mu^2 = 1 + \frac{x^2}{4 \cdot 5} + \frac{x^4}{4 \cdot 5 \cdot 6 \cdot 7} + \ldots,$$

we have after reduction

$$x^3 + \frac{6(E - 1)}{E} \cdot \frac{1}{\mu^2} \cdot x - \frac{6N_0}{E} \cdot \frac{1}{\mu^2} = 0,$$

a cubic equation, which may be solved by means of the following formulæ:—

$$\begin{aligned} &\alpha_0 = 2(E - 1)/E, \quad \alpha_1 = E/3N_0; \\ &\alpha = \alpha_0 \alpha_1^{\frac{3}{2}}, \qquad \beta = 2\alpha_0^{\frac{1}{2}}; \\ &\tan \theta = \alpha/\mu, \quad \tan \varphi = \tan^{\frac{1}{3}} \tfrac{1}{2}\theta, \quad x = \beta/\mu \tan \varphi. \end{aligned}$$

μ is obtained from the accompanying table with the argument x^2. In the first approximation, unless otherwise known, μ may be assumed equal to unity.

x^2	log μ	Diff.	x^2	log μ	Diff.	x^2	log μ	Diff.	x^2	log μ	Diff.
0.0	0.00000		1.0	0.01084		2.0	0.02166		3.0	0.03245	
0.1	0.00109	109	1.1	0.01193	109	2.1	0 02274	108	3.1	0.03353	108
0.2	0.00217	108	1.2	0.01301	108	2.2	0.02382	108	3.2	0.03460	107
0.3	0.00326	109	1.3	0.01409	108	2.3	0.02490	108	3.3	0.03568	108
0.4	0.00434	108	1.4	0.01517	108	2.4	0.02598	108	3.4	0.03676	108
0.5	0.00543	109	1.5	0.01626	109	2.5	0 02706	108	3.5	0.03783	107
0.6	0.00651	108	1.6	0.01734	108	2.6	0.02814	108	3.6	0.03891	108
0.7	0.00759	108	1.7	0.01842	108	2.7	0.02922	108	3.7	0.03998	107
0.8	0.00868	109	1.8	0.01950	108	2.8	0.03029	107	3.8	0.04106	108
0.9	0.00976	108	1.9	0.02058	108	2.9	0.03137	108	3.9	0.04213	107
1.0	0.01084	108	2.0	0.02166	108	3.0	0.03245	108	4.0	0.04321	108

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SOLUTIONS OF EXERCISES.

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THE dome of the rotunda of the University of Virginia is spherical. The length of its meridian section is $85^f.2$ and the girth of its base is 214^f. From these data it is required to compute the radius and the surface of the dome.

[*William M. Thornton.*]

SOLUTION.

We have for the chord of the meridian arc

$$\frac{214}{\pi} = 68.1.$$

If therefore x be the half central angle of the arc,

$$\frac{\sin x}{x} = \frac{681}{852} = 0.799.$$

Passing to logarithms and expressing x in minutes by putting

$$x = \mathrm{x} \cdot \frac{\pi}{10800},$$

we get $$y = \log \sin x - \log \mathrm{x} + 3.6336 = 0.$$

By trial we find, if

$$x = 64^\circ, \quad y = +\,0.0030;$$
$$x = 65^\circ, \quad y = -\,0.0002;$$